GPLUS EDUCATION

Date :
Time : **MATHEMATICS**
Marks :

CONTINUITY AND DIFFERENTIABILITY

Single Correct Answer Type

1. If $y = \cos^{-1}\cos(|x| - f(x))$, where
$f(x) = \begin{cases} 1, if\ x > 0 \\ -1, if\ x < 0 \\ 0, if\ x = 0 \end{cases}$. Then, $(dy/dx)\ x = \frac{5\pi}{4}$ is equal to
a) -1 b) 1
c) 0 d) Cannot be determined

2. If $f(x) = \begin{cases} \frac{1-\sqrt{2}\sin x}{\pi - 4x}, \text{if } x \neq \frac{\pi}{4} \\ a, \quad \text{if } x = \frac{\pi}{4} \end{cases}$ is continuous at $\frac{\pi}{4}$, then a is equal to
a) 4 b) 2 c) 1 d) 1/4

3. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then
a) $f(6) = 5$ b) $f(6) < 5$ c) $f(6) < 8$ d) $f(6) \geq 8$

4. If for a continuous function f, $f(0) = f(1) = 0, f'(1) = 2$ and $y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to
a) 1 b) 2 c) 0 d) None of these

5. If $f(x) = \begin{cases} [\cos \pi\ x], x < 1 \\ |x - 2|,\ 2 > x \geq 1 \end{cases}$, then $f(x)$ is
a) Discontinuous and non-differentiable at $x = -1$ and $x = 1$
b) Continuous and differentiable at $x = 0$
c) Discontinuous at $x = 1/2$
d) Continuous but not differentiable at $x = 2$

6. Given that $f(x)$ is a differentiable function of x and that $f(x).f(y) = f(x) + f(y) + f(xy) - 2$ and that $f(2) = 5$. Then, $f(3)$ is equal to
a) 10 b) 24 c) 15 d) None of these

7. If $f(x)$ is continuous at $x = 0$ and $f(0) = 2$, then
$\lim_{x \to 0} \frac{\int_0^x f(u)du}{x}$ is
a) 0 b) 2 c) $f(2)$ d) None of these

8. The function $f(x) = \begin{cases} \frac{\tan x}{x},\ x \neq 0 \\ 1,\ x = 0 \end{cases}$, is
a) Continuous but not differentiable at $x = 0$ b) Discontinuous at $x = 0$
c) Continuous and differentiable at $x = 0$ d) Not defined at $x = 0$

9. If $f(x + y + z) = f(x).f(y).f(z)$ for all x, y, z and $f(2) = 4, f'(0) = 3$, then $f'(2)$ equals
a) 12 b) 9 c) 16 d) 6

10. If $f(x) = \begin{cases} \frac{x-1}{2x^2 - 7x + 5},\ \text{for } x \neq 1 \\ -\frac{1}{3},\ \text{for } x = 1 \end{cases}$, then $f'(1)$ is equal to
a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) -13 d) 1/3

11. Let $f(x)$ be an odd function. Then $f'(x)$
a) Is an even function b) Is an odd function c) May be even or odd d) None of these

12. If $f(x + y) = f(x)f(y)$ for all $x, y \in R, f(5) = 2, f'(0) = 3$. Then $f'(5)$ equals
a) 6 b) 3 c) 5 d) None of these

13. The function f defined by
$f(x)=\begin{cases}\frac{\sin x^2}{x}, x\neq 0\\ 0,\ x=0\end{cases}$ is
a) Continuous and derivable at $x=0$
b) Neither continuous nor derivable at $x=0$
c) Continuous but not derivable at $x=0$
d) None of these
14. Given $f(0)=0$ and $f(x)=\frac{1}{(1-e^{-1/x})}$ for$x\neq 0$. Then only one of the follo0wing statements on $f(x)$ is true. That id $f(x)$, is
a) Continuous at $x=0$
b) Not continuous at $x=0$
c) Both continuous and differentiable at $x=0$
d) Not defined at $x=0$
15. Let $f(x)=\begin{cases}x^P\sin\frac{1}{x}, x\neq 0\\ 0, \quad x=0\end{cases}$ then $f(x)$ is continuous but not differentiable at $x=0$, if
a) $0<p\leq 1$ b) $1\leq p<\infty$ c) $-\infty<p<0$ d) $p=0$
16. If $f(x)$ defined by $f(x)=\begin{cases}\frac{|x^2-x|}{x^2-x}, x\neq 0,1\\ 1,\ x=0\\ -1,\ x=1\end{cases}$ then $f(x)$ is continuous for all
a) x
b) x except at $x=0$
c) x except at $x=1$
d) x except at $x=0$ and $x=1$
17. If for a function $f(x), f(2)=3, f'(2)=4$, then $\lim_{x\to 2}[f(x)]$, where $[\cdot]$ denotes the greatest integer function, is
a) 2 b) 3 c) 4 d) Non-existent
18. The function $f(x)=\frac{1-\sin x+\cos x}{1+\sin x+\cos x}$ is not defined at$x=\pi$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x=\pi$, is
a) -1/2 b) ½ c) -1 d) 1
19. If $f(x)=[\sqrt{2}\sin x]$, where $[x]$ represents the greatest integer function, then
a) $f(x)$ is periodic
b) Maximum value of $f(x)$ is 1 in the interval $[-2\pi, 2\pi]$
c) $f(x)$ is discontinuous at $x=\frac{n\pi}{2}+\frac{\pi}{4}, n\in Z$
d) $f(x)$ is differentiable at $x=n\pi, n\in Z$
20. If $f(x)=|x^2-4x+3|$, then
a) $f'(1)=-1$ and $f'(3)=1$
b) $f'(1)=-1$ and $f'(3)$ does not exist
c) $f'(1)=-1$ does not exist and $f'(3)=1$
d) Both $f'(1)$ and $f'(3)$ do not exist
21. The function $f(x)=\begin{cases}\frac{e^{1/x}-1}{e^{1/x}+1}, x\neq 0\\ 0,\ x=0\end{cases}$
a) Is continuous at $x=0$
b) Is not continuous at $x=0$
c) Is not continuous at $x=0$, but can be made continuous $x=0$
d) None of these
22. If $f(x)=x^2+\frac{x^2}{1+x^2}+\frac{x^2}{(1+x^2)^2}+\cdots+\frac{x^2}{(1+x^2)^n}+\cdots$, then at $x=0$, $f(x)$

a) Has no limit
b) Is discontinuous
c) Is continuous but not differentiable
d) Is differentiable

23. if $f(x) = a|\sin x| + b\, e^{|x|} + c\,|x|^3$ and if $f(x)$ is differentiable at $x = 0$, then
a) $a = b = c = 0$ b) $a = 0, b = 0; c \in R$ c) $b = c = 0, a \in R$ d) $c = 0, a = 0, b \in R$

24. At the point $x = 1$, the function $f(x) = \begin{cases} x^3 - 1, & 1 < x < \infty \\ x - 1, & -\infty < x \leq 1 \end{cases}$
a) Continuous and differentiable
b) Continuous and not differentiable
c) Discontinuous and differentiable
d) Discontinuous and not differentiable

25. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is
a) Continuous as well as differentiable for all x
b) Continuous for all x but not differentiable at $x = 0$
c) Neither differentiable nor continuous at $x = 0$
d) Discontinuous everywhere

26. If $f(x) = x(\sqrt{x} + \sqrt{x+1})$, then
a) $f(x)$ is continuous but not differentiable at $x = 0$ b) $f(x)$ is differentiable at $x = 0$
c) $f(x)$ is not differentiable at $x = 0$ d) None of the above

27. If $f(x) = \begin{cases} ax^2 + b, & b \neq 0, x \leq 1 \\ x^2 b + ax + c, & x > 1 \end{cases}$, then, $f(x)$ is continuous and differentiable at $x = 1$, if
a) $c = 0, a = 2b$ b) $a = b, c \in R$ c) $a = b, c = 0$ d) $a = b, c \neq 0$

28. If $f(x) = |\log_{10} x|$, then at $x = 1$
a) $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = -\log_{10} e$
b) $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = \log_{10} e$
c) $f(x)$ is continuous and $f'(1^-) = \log_{10} e$, $f'(1^+) = -\log_{10} e$
d) None of these

29. Let $f(x) = [x] + \sqrt{x - [x]}$, where $[x]$ denotes the greatest integer function. Then,
a) $f(x)$ is continuous on R^+
b) $f(x)$ is continuous on R
c) $f(x)$ is continuous on $R - Z$
d) None of these

30. Let $f(x)$ be a function differentiable at $x = c$. Then, $\lim_{x \to c} f(x)$ equals
a) $f'(c)$ b) $f''(c)$ c) $\frac{1}{f(c)}$ d) None of these

31. If $f(x + y) = f(x)f(y)$ for all real x and y, $f(6) = 3$ and $f'(0) = 10$, then $f'(6)$ is
a) 30 b) 13 c) 10 d) 0

32. Let $f(x)$ be twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$, $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then $h(10)$ is equal to
a) 22 b) 11 c) 0 d) None of these

33. If $y = f(x) = \frac{1}{u^2+u-1}$ where $u = \frac{1}{x-1}$, then the function is discontinuous at $x =$
a) 1 b) 1/2 c) 2 d) -2

34. For the function $f(x) = \begin{cases} |x - 3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ which one of the following is incorrect?
a) Continuous at $x = 1$ b) Derivable at $x = 1$ c) Continuous at $x = 3$ d) Derivable at $x = 3$

35. For the function $f(x)=\frac{\log_e(1+x)+\log_e(1-x)}{x}$ to be continuous at $=0$, the value of $f(0)$ is

a) -1 b) 0 c) -2 d) 2

36. If $f(x)=|x-a|\phi(x)$, where $\phi(x)$ is continuous function, then

a) $f'(a^+)=\phi(a)$ b) $f'(a^-)=\phi(a)$ c) $f'(a^+)=f'(a^-)$ d) None of these

37. At $x=0$, the function $f(x)=|x|$ is

a) Continuous but not differentiable
b) Discontinuous and differentiable
c) Discontinuous and not differentiable
d) Continuous and differentiable

38. If $f: R \to R$ is defined by

$$f(x)=\begin{cases}\dfrac{2\sin x-\sin 2x}{2x\cos x}, & \text{if } x\neq 0,\\ a, & \text{if } x=0\end{cases}$$

Then the value of a so that f is continuous at 0 is

a) 2 b) 1 c) -1 d) 0

39. The function $f(x)=|x|+\frac{|x|}{x}$ is

a) Discontinuous at origin because $|x|$ is discontinuous there
b) Continuous at origin
c) Discontinuous at origin because both $|x|$ and $\frac{|x|}{x}$ are discontinuous there
d) Discontinuous at the origin because $\frac{|x|}{x}$ is discontinuous there

40. $f(x)=|x-3|$ is ... at $x=3$

a) Continuous and not differentiable
b) Continuous and differentiable
c) Discontinuous and not differentiable
d) Discontinuous and differentiable

41. The points of discontinuity of $\tan x$ are

a) $n\pi, n\in I$ b) $2n\pi, n\in I$ c) $(2n+1)\frac{\pi}{2}, n\in I$ d) None of these

42. The number of points at which the function $f(x)=|x-0.5|+|x-1|+\tan x$ does not have a derivative in the interval (0, 2), is

a) 1 b) 2 c) 3 d) 4

43. $f(x)=\begin{cases}2a-x & \text{in } -a<x<a\\ 3x-2a & \text{in } a\leq x\end{cases}$. Then, which of the following is true?

a) $f(x)$ is discontinuous at $x=a$
b) $f(x)$ is not differentiable at $x=a$
c) $f(x)$ is differentiable at $x\geq a$
d) $f(x)$ is continuous at all $x<a$

44. If the derivative of the function $f(x)$ is everywhere continuous and is given by

$f(x)=\begin{cases}bx^2+ax+4; & x\geq -1\\ ax^2+b; & x<-1\end{cases}$, then

a) $a=2, b=-3$ b) $a=3, b=2$ c) $a=-2, b=-3$ d) $a=-3, b=-2$

45. The function $f(x)=\begin{cases}|x-3|, & \text{if } x\geq 1\\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}, & \text{if } x<1\end{cases}$ is

a) Continuous and differentiable at $x=3$
b) Continuous at $x=3$, but not differentiable at $x=3$
c) continuous and differentiable everywhere
d) continuous at $x=1$, but not differentiable at $x=1$

46. If the function $f: R\to R$ given by $f(x)=\begin{cases}x+a, & \text{if } x\leq 1\\ 3-x^2, & \text{if } x>1\end{cases}$ is continuous at $x=1$, thyen a is equal to

a) 4 b) 3 c) 2 d) 1

47. Let a function $f: R\to R$, where R is the set of real numbers satisfying the equation $f(x+y)=f(x)+f(y), \forall\, x, y$ if $f(x)$ is continuous at $x=0$, then

a) $f(x)$ is discontinuous, $\forall\, x\in R$
b) $f(x)$ is continuous, $\forall\, x\in R$
c) $f(x)$ is continuous for $x\in\{1,2,3,4\}$
d) None of the above

48. Let function $f(x) = \sin^{-1}(\cos x)$, is
a) Discontinuous at $x = 0$
b) Continuous at $x = 0$
c) Differentiable at $x = 0$
d) None of these

49. If $f(x) = \begin{cases} (x-2)^2 \sin\left(\frac{1}{x-2}\right) - |x-1|, & x \neq 2 \\ -1, & x = 2 \end{cases}$ then the set of points where $f(x)$ is differentiable, is
a) R
b) $R - \{1, 2\}$
c) $R - \{1\}$
d) $R - \{2\}$

50. If $f(x) = \begin{cases} |x-4|, \text{for } x \geq 1 \\ (x^3/2) - x^2 + 3x + (1/2), \text{for } x < 1 \end{cases}$, then
a) $f(x)$ is continuous at $x = 1$ and $x = 4$
b) $f(x)$ is differentiable at $x = 4$
c) $f(x)$ is continuous and differentiable at $x = 1$
d) $f(x)$ is not continuous at $x = 1$

51. Let $f(x+y) = f(x) + f(y)$ and $f(x) = x^2\, g(x)$ for all $x, y \in R$, where $g(x)$ is continuous function. Then, $f'(x)$ is equal to
a) $g'(x)$
b) $g(0)$
c) $g(0) + g'(x)$
d) 0

52. If the function
$$f(x) = \begin{cases} \{1 + |\sin x|\}^{\frac{a}{|\sin x|}}, & -\frac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \end{cases}$$
Is continuous at $x = 0$
a) $a = \log_e b\,, b = \frac{2}{3}$
b) $b = \log_e a\,, a = \frac{2}{3}$
c) $a = \log_e b\,, b = 2$
d) None of these

53. If $f: R \to R$ is defined by
$f(x) = \begin{cases} \frac{\cos 3x - \cos x}{x^2}, & \text{for } x \neq 0 \\ \lambda, & \text{for } x = 0 \end{cases}$ and if f is continuous at $x = 0$, then λ is equal to
a) -2
b) -4
c) -6
d) -8

54. $\lim_{x \to 0} [(1+3x)^{1/x}] = k$, then for continuity at $x = 0, k$ is
a) 3
b) -3
c) e^3
d) e^{-3}

55. If $f(x) = x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)^2} + \cdots$ to ∞ then at $x = 0, f(x)$ is
a) Continuous but not differentiable
b) Differentiable
c) Continuous
d) None of these

56. The set of points of discontinuity of the function
$f(x) = \lim_{n \to \infty} \frac{x^{-n} - x^n}{x^{-n} + x^n}, n \in Z$ is
a) $\{1\}$
b) $\{-1\}$
c) $\{-1, 1\}$
d) None of these

57. Let $f(x) = |x| + |x-1|$, then
a) $f(x)$ is continuous at $x = 0$, as well as at $x = 1$
b) $f(x)$ is continuous at $x = 0$, but not at $x = 1$
c) $f(x)$ is continuous at $x = 1$, but not at $x = 0$
d) None of these

58. The value of derivative of $|x-1| + |x-3|$ at $x = 2$, is
a) -2
b) 0
c) 2
d) Not defined

59. If $f(x) = \text{Min}\,\{\tan x\,, \cot x\}$, then

a) $f(x)$ is not differentiable at $x = 0, \pi/4, 5\pi/4$
b) $f(x)$ is continuous at $x = 0, \pi/2, 3\pi/2$
c) $\int_0^{\pi/2} f(x)dx = \ln\sqrt{2}$
d) $f(x)$ is periodic with period $\frac{\pi}{2}$

60. On the interval $I = [-2, 2]$, the function $f(x) = \begin{cases}(x+1)\, e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0\end{cases}$
a) Is continuous for all $x \in I - \{0\}$
b) Assumes all intermediate values from $f(-2)$ to $f(2)$
c) Has a maximum value equal to $3/e$
d) All the above

61. The value of $f(0)$, so that the function $f(x) = \frac{(27-2x)^{1/3}-3}{9-3(243+5\,x)^{1/5}}(x \neq 0)$ is continuous is given by

a) $\frac{2}{3}$ b) 6 c) 2 d) 4

62. The function $f(x) = \begin{cases}1 - 2x + 3x^2 - 4x^3 + \cdots \text{to } \infty, & x \neq -1 \\ 1, & x = -1\end{cases}$ is
a) Continuous and derivable at $x = -1$
b) Neither continuous nor derivable at $x = -1$
c) Continuous but not derivable at $x = -1$
d) None of these

63. If $f(x) = \begin{cases}\frac{|x+2|}{\tan^{-1}(x+2)}, & x \neq -2 \\ 2, & x = -2\end{cases}$, then $f(x)$ is
a) Continuous at $x = -2$
b) Not continuous $x = -2$
c) Differentiable at $x = -2$
d) Continuous but not derivable at $x = -2$

64. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h) = 5$, then $f'(1)$ equals

a) 6 b) 5 c) 4 d) 3

65. If $\lim_{x\to a^+} f(x) = l = \lim_{x\to a^-} g(x)$ and $\lim_{x\to a^-} f(x) = m \lim_{x\to a^+} g(x)$, then the function $f(x)\, g(x)$
a) Is not continuous at $x = a$
b) Has a limit when $x \to a$ and it is equal to lm
c) Is continuous at $x = a$
d) Has a limit when $x \to a$ but it is not equal to lm

66. A function $f(x) = \begin{cases}1 + x, & x \leq 2 \\ 5 - x, & x > 2\end{cases}$ is

a) Not continuous at $x = 2$ b) Differenti8able at $x = 2$
c) Continuous but not differentiable at $= 2$ d) None of the above

67. If $f(x) = \begin{cases}x\sin\frac{1}{x}, & x \neq 0 \\ k, & x = 0\end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1 b) -1 c) 0 d) 2

68. Let $f(x) = \begin{cases}x^n \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0\end{cases}$. Then, $f(x)$ is continuous but not differentiable at $x = 0$, if

a) $n \in (0, 1]$ b) $n \in [1, \infty)$ c) $n \in (-\infty, 0)$ d) $n = 0$

69. ln[1, 3] the function $[x^2 + 1]$, $[x]$ denoting the greatest integer function, is continuous
a) For all x
b) For all x except at four points

c) For all except at seven points
d) For all except at eight-points

70. If $f(x)=\begin{cases}\frac{x\log\cos x}{\log(1+x^2)}, & x\neq 0\\ 0, & x=0\end{cases}$, then
a) $f(x)$ is not continuous at $x=0$
b) $f(x)$ is not continuous and differentiable at $x=0$
c) $f(x)$ is not continuous at $x=0$ but not differentiable at $x=0$
d) None of these

71. Let $f(x)=\begin{cases}5^{1/x}, & x<0\\ \lambda[x], & x\geq 0\end{cases}$ and $\lambda\in R$, then at $x=0$
a) f is discontinuous
b) f is continuous only, if $\lambda=0$
c) f is continuous only, whatever λ may be
d) None of the above

72. The function $f(x)=a[x+1]+b[x-1]$, where $[x]$ is the greatest integer function, is continuous at $x=1$, is
a) $a+b=0$
b) $a=b$
c) $2a-b=0$
d) None of these

73. If $f(x)=\int_{-1}^{x}|t|\,dt, x\geq -1$, then
a) f and f' are continuous for $x+1>0$
b) f is continuous but f' is not so for $x+1>0$
c) f and f' are continuous at $x=0$
d) f is continuous at $x=0$ but f' is not so

74. Let $f(x)=\frac{1-\tan x}{4x-\pi}, x\neq\frac{\pi}{4},\ x\in\left[0,\frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0,\frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is
a) 1
b) 1/2
c) -1/2
d) -1

75. If $f(x)=\begin{cases}\log_{(1-3x)}(1+3x), & \text{for } x\neq 0\\ k, & \text{for } x=0\end{cases}$ is continuous at $x=0$, then k is equal to
a) -2
b) 2
c) 1
d) -1

76. Let $f(x)=|x|$ and $g(x)=|x^3|$, then
a) $f(x)$ and $g(x)$ Both are continuous at $x=0$
b) $f(x)$ and $g(x)$ Both are differentiable at $x=0$
c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x=0$
d) $f(x)$ and $g(x)$ Both are not differentiable at $x=0$

77. If the function $f(x)=\begin{cases}Ax-B, & x\leq 1\\ 3x, & 1<x<2\\ B\,x^2-A, & x\geq 2\end{cases}$ be continuous at $x=1$ and discontinuous at $x=2$, then
a) $A=3+B, B\neq 3$
b) $A=3+B, B=3$
c) $A=3+B$
d) None of these

78. Let a function $f(x)$ be defined by $f(x)=\begin{cases}x, & x\in Q\\ 0, & x\in R-Q\end{cases}$ Then, $f(x)$ is
a) Everywhere continuous
b) Nowhere continuous
c) Continuous only at some points
d) Discontinuous only at some points

79. Which one of the following is not true always?
a) If $f(x)$ is not continuous at $x=a$, then it is not differentiable at $x=a$
b) If $f(x)$ is continuous at $x=a$, then it is differentiable at $x=a$
c) If $f(x)$ and $g(x)$ are differentiable at $x=a$, then $f(x)+g(x)$ is also differentiable at $x=a$
d) If a function $f(x)$ is continuous at $x=a$, then $\lim_{x\to a}f(x)$ exists

80. Suppose a function $f(x)$ satisfies the following conditions for all x and y: (i) $f(x+y)=f(x)f(y)$ (ii) $f(x)=1+x\,g(x)\log a$, where $a>1$ and $\lim_{x\to 0}g(x)=1$. Then, $f'(x)$ is equal to
a) $\log a$
b) $\log a^{f(x)}$
c) $\log(f(x))^a$
d) None of these

81. If $f(x) = |\log_e x|$, then
a) $f'(1^+) = 1, f'(1^-) = -1$
b) $f'(1^-) = -1, f'(1^+) = 0$
c) $f'(1) = 1, f'(1^-) = 0$
d) $f'(1) = -1, f'(1^+) = -1$
82. The derivative of $f(x) = |x|^3$ at $x = 0$ is
a) -1 b) 0 c) Does not exist d) None of these
83. Let $f(x) = \begin{cases} \frac{x^4-5x^2+4}{|(x-1)(x-2)|}, x \neq 1, 2 \\ 6, \; x = 10 \\ 12, \; x = 2 \end{cases}$ Then, $f(x)$ is continuous on the set
a) R b) $R - \{1\}$ c) $R - \{2\}$ d) $R - \{1, 2\}$
84. Let $f(x + y) = f(x)f(y)$ for all $x, y \in R$. If $f'(1) = 2$ and $f(4) = 4$, then $f'(4)$ equal to
a) 4 b) 1 c) 1/2 d) 8
85. If $f(x) = (x - x_0)\phi(x)$ and $\phi(x)$ is continuous at $x = x_0$, then $f'(x_0)$ is equal to
a) $\phi'(x_0)$ b) $\phi(x_0)$ c) $x_0\phi(x_0)$ d) None of these
86. The function $f(x) = \begin{cases} x^n \sin\left(\frac{1}{x}\right), x \neq 0 \\ 0, \; x = 0 \end{cases}$ is continuous and differentiable at $x = 0$, if
a) $n \in (0, 1]$ b) $n \in [1, \infty)$ c) $n \in (1, \infty)$ d) $n \in (-\infty, 0)$
87. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, x < 4 \\ a + b, \quad x = 4 \\ \frac{x-4}{|x-4|} + b, x > 4 \end{cases}$
Then, $f(x)$ is continuous at $x = 4$ when
a) $a = 0, b = 0$ b) $a = 1, b = 1$ c) $a = -1, b = 1$ d) $a = 1, b = -1$
88. Let $f: R \to R$ be a function defined by$f(x) = \min\{x + 1, |x| + 1\}$. Then, which of the following is true?
a) $f(x) \geq 1$ for all $x \in R$
b) $f(x)$ is not differentiable at $x = 1$
c) $f(x)$ is differentiable everywhere
d) $f(x)$ is not differentiable at $x = 0$
89. Let$f(x) = \begin{cases} 1, & \forall \quad x < 0 \\ 1 + \sin x, & \forall \quad 0 \leq x \leq \pi/2 \end{cases}$, then what is the value of $f'(x)$ at $x = 0$?
a) 1 b) -1 c) ∞ d) Does not exist
90. If $f(x) = \frac{1}{2} x - 1$, then on the interval $[0, \pi]$,
a) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous
b) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous
c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous
d) $\tan[f(x)]$ s continuous but $\frac{1}{f(x)}$ is not
91. At $x = \frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is
a) Continuous b) Discontinuous c) Differentiable d) Non-zero
92. Let $f(x)$ be defined on R such that $f(1) = 2, f(2) = 8$ and $f(u + v) = f(u) + kuv - 2v^2$ for all $u, v \in R$ (k is a fixed constant). Then,
a) $f'(x) = 8x$ b) $f(x) = 8x$ c) $f'(x) = x$ d) None of these
93. The number of discontinuities of the greatest integer function $f(x) = [x], x \in \left(-\frac{7}{2}, 100\right)$ is equal to
a) 104 b) 100 c) 102 d) 103
94. If $f(x) = |\log_e |x||$, then $f'(x)$ equals
a) $\frac{1}{|x|}, x \neq 0$
b) $\frac{1}{x}$ for $|x| > 1$ and $\frac{-1}{x}$ for $|x| < 1$

c) $\frac{-1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

d) $\frac{1}{x}$ for $|x| > 0$ and $-\frac{1}{x}$ for $x < 0$

95. Function $f(x) = \begin{cases} x-1, x < 2 \\ 2x-3, x \geq 2 \end{cases}$ is a continuous function

a) For $x = 2$ only
b) For all real values of x such that $x \neq 2$
c) For all real values of x
d) For all integer values of x only

96. The value of $f(0)$ so that the function $f(x) = \frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2}$ $(x \neq 0)$ is continuous everywhere, is given by

a) -1
b) 1
c) 26
d) None of these

97. The function $f(x) \begin{cases} (x+1)^{2-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, x \neq 0 \\ 0, \quad x = 0 \end{cases}$ is

a) Continuous everywhere
b) Discontinuous at only one point
c) Discontinuous at exactly two points
d) None of these

98. If $f(x) = \sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}$, then $f(x)$ is differentiable on

a) $(-\infty, \infty)$
b) $[2, \infty) - \{4\}$
c) $[2, \infty)$
d) None of these

99. $f(x) = \sin|x|$. Then $f(x)$ is not differentiable at

a) $x = 0$ only
b) All x
c) Multiples of π
d) Multiples of $\frac{\pi}{2}$

100. If $f(x) = \begin{cases} mx+1, \ x \leq \frac{\pi}{2} \\ \sin x + n, \ x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

a) $m = 1, n = 0$
b) $m = \frac{n\pi}{2} + 1$
c) $n = \frac{m\pi}{2}$
d) $m = n = \frac{\pi}{2}$

101. If $f: R \to R$ is defined by

$f(x) = \begin{cases} \frac{x+2}{x^2+3x+2}, & \text{if } x \in R - \{-1, -2\} \\ -1, & \text{if } x = -2 \\ 0, & \text{if } x = -1 \end{cases}$, then f is continuous on the set

a) R
b) $R - \{-2\}$
c) $R - \{-1\}$
d) $R - (-1, -2)$

102. Let $f(x+y) = f(x)f(y)$ for all $x, y \in R$. Suppose that $f(3) = 3$ and $f'(0) = 11$ then, $f'(3)$ is equal to

a) 22
b) 44
c) 28
d) None of these

103. If $f(x) = \begin{cases} \frac{1-\sin x}{(\pi-2x)^2} \cdot \frac{\log \sin x}{(\log 1+\pi^2-4\pi x+x^2)}, x \neq \frac{\pi}{2} \\ k, \ x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \pi/2$, then $k =$

a) $-\frac{1}{16}$
b) $-\frac{1}{32}$
c) $-\frac{1}{64}$
d) $-\frac{1}{28}$

104. The set of points where the function $f(x) = x|x|$ is differentiable is

a) $(-\infty, \infty)$
b) $(-\infty, 0) \cup (0, \infty)$
c) $(0, \infty)$
d) $[0, \infty)$

105. The value of $f(0)$ so that $\frac{(-e^x+2^x)}{x}$ may be continuous at $x = 0$ is

a) $\log\left(\frac{1}{2}\right)$
b) 0
c) 4
d) $-1 + \log 2$

106. For the function $f(x) = \begin{cases} \frac{x^3-a^3}{x-a}, x \neq a \\ b, \quad x = a \end{cases}$, if $f(x)$ is continuous at $x = a$, then b is equal to

a) a^2
b) $2a^2$
c) $3a^2$
d) $4a^2$

107. $f(x) = x + |x|$ is continuous for

a) $x \in (-\infty, \infty)$
b) $x \in (-\infty, \infty) - \{0\}$
c) Only $x > 0$
d) No value of x

108. If $f(x) \begin{cases} \frac{[x]-1}{x-1}, x \neq 1 \\ 0, \quad x = 1 \end{cases}$ then at $x = 1$, $f(x)$ is

a) Continuous and differentiable
b) Differentiable but not continuous
c) Continuous but not differentiable
d) Neither continuous nor differentiable

109. If $f: R \to R$ given by

$$f(x) = \begin{cases} 2\cos x, \text{if } x \leq -\frac{\pi}{2} \\ a + \sin x + b, \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1 + \cos^2 x, \text{ if } x \geq \frac{\pi}{2} \end{cases}$$ is a continuous

Function on R, then (a, b) is equal to

a) (1/2, 1/2) b) (0, -1) c) (0, 2) d) (1, 0)

110. If $f(x) = [x \sin \pi \; x]$, then which of the following is incorrect?

a) $f(x)$ is continuous at $x = 0$
b) $f(x)$ is continuous in $(-1, 0)$
c) $f(x)$ is differentiable at $x = 1$
d) $f(x)$ is differentiable in $(-1, 1)$

111. The function defined by

$$f(x) = \begin{cases} \left(x^2 + e^{\frac{1}{2-x}}\right)^{-1} & x \neq 2 \\ k, & x = 2 \end{cases}$$ is continuous from right at the point $x = 2$, then k is equal to

a) 0 b) $\frac{1}{4}$ c) $-\frac{1}{2}$ d) None of these

112. The number of points of discontinuity of the function

$f(x) = \frac{1}{\log|x|}$, is

a) 4 b) 3 c) 2 d) 1

113. Let $f(x) = (x + |x|)|x|$. The, for all x

a) f and f' are continuous
b) f is differentiable for some x
c) f' is not continuous
d) f'' is continuous

114. If $f(x) = \begin{cases} 1, x < 0 \\ 1 + \sin x, 0 \leq x \leq \frac{\pi}{2} \end{cases}$ then derivative of $f(x)$ at $x = 0$

a) Is equal to 1 b) Is equal to 0 c) Is equal to -1 d) Does not exist

115. Function $f(x) = |x - 1| + |x - 2|, x \in R$ is

a) Differentiable everywhere in R
b) Except $x = 1$ and $x = 2$ differentiable everywhere in R
c) Not continuous at $x = 1$ and $x = 2$
d) Increasing in R

116. The function $f(x)$ is defined as $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$, if $x \neq 0$. The value of f to be assigned at $x = 0$ so that the function is continuous there, is

a) $-\frac{1}{3}$ b) 1 c) $\frac{2}{3}$ d) $\frac{1}{3}$

117. If $f(x) = \begin{cases} x, & \text{for} \quad 0 < x < 1 \\ 2 - x, & \text{for} \quad 1 \leq x < 2 \\ x - (1/2)x^2, & \text{for} \;\; x = 2 \end{cases}$. Then, $f'(1)$ is equal to

a) -1 b) 1 c) 0 d) None of these

118. The function $f(x) = \begin{cases} 1, \quad |x| \geq 1 \\ \frac{1}{n^2}, \frac{1}{n} < |x| < \frac{1}{n-1}, n = 2, 3, \ldots \\ 0, \quad x = 0 \end{cases}$

a) Is discontinuous at finitely many points
b) Is continuous everywhere
c) Is discontinuous only at $x = \pm\frac{1}{n}, n \in Z - \{0\}$ and $x = 0$
d) None of these

119. The set of points where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable is
a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(-1, \infty)$ d) None of these

120. if $f(x) = |x|^3$, then $f'(0)$ equals
a) 0 b) 1/2 c) -1 d) -1/2

121. If function $f(x) = \begin{cases} x, \text{if } x \text{ is rational} \\ 1 - x, \text{ if } x \text{ is irrational} \end{cases}$, then the number of points at which $f(x)$ is continuous, is
a) ∞ b) 1 c) 0 d) None of these

122. If $f(x) = \begin{cases} \frac{2^x - 1}{\sqrt{1+x} - 1}, -1 \le x < \infty, \ x \ne 0 \\ k, \qquad x = 0 \end{cases}$ is continuous everywhere, then k is equal to
a) $\frac{1}{2}\log_e 2$ b) $\log_e 4$ c) $\log_e 8$ d) $\log_e 2$

123. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f(x)$ is differentiable on
a) $[-1, 1]$ b) $R - \{-1, 1\}$ c) $R - (-1, 1)$ d) None of these

124. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^3}$. Then, $g'(x)$ is equal to
a) $\frac{1}{1 + (g(x))^3}$ b) $\frac{1}{1 + (f(x))^3}$ c) $1 + (g(x))^3$ d) $1 + (f(x))^3$

125. If$f(x) = ae^{|x|} + b|x|^2$; $a, b \in R$ and $f(x)$ is differentiable at $x = 0$. Then a and b are
a) $a = 0, b \in R$ b) $a = 1, b = 2$ c) $b = 0, a \in R$ d) $a = 4, b = 5$

126. The set of points of differentiability of the function $f(x) = \begin{cases} \frac{\sqrt{x+1} - 1}{x}, \text{for } x \ne 0 \\ 0, \text{for } x = 0 \end{cases}$ is
a) R b) $[0, \infty]$ c) $(-\infty, 0)$ d) $R - \{0\}$

127. Let $f(x) = ||x| - 1|$, then points where $f(x)$ is not differentiable, is/(are)
a) $0, \pm 1$ b) ± 1 c) 0 d) 1

128. If the function $f(x) = \begin{cases} \frac{1 - \cos x}{x^2}, \ \text{for } x \ne 0 \\ k, \ \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is
a) 1 b) 0 c) $\frac{1}{2}$ d) -1

129. If $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, x < 0 \\ c, \ x = 0 \\ \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx\sqrt{x}}, x > 0 \end{cases}$ is continuous at $x = 0$, then
a) $a = -\frac{3}{2}, b = 0, c = \frac{1}{2}$
b) $a = -\frac{3}{2}, b = 1, c = -\frac{1}{2}$
c) $a = -\frac{3}{2}, b \in R - \{0\}, c = \frac{1}{2}$
d) None of these

130. If $f(x) = (x + 1)^{\cot x}$ be continuous at $= 0$, then $f(0)$ is equal to
a) 0 b) $-e$ c) e d) None of these

131. Let f is a real-valued differentiable function satisfying $|f(x) - f(y)| \le (x - y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals
a) 1 b) 2 c) 0 d) -1

132. The function $f(x) = x - |x - x^2|, -1 \le x \le 1$ is continuous on the interval

a) $[-1, 1]$ b) $(-1, 1)$ c) $[-1, 0) \cup (0, 1]$ d) $(-1, 0) \cup (0, 1)$

133. The function $f(x) = x - |x - x^2|$ is
a) Continuous at $x = 1$
b) Discontinuous at $x = 1$
c) Not defined at $x = 1$
d) None of the above

134. The set of points where the function $f(x) = |x - 1|e^x$ is differentiable, is
a) R b) $R - \{1\}$ c) $R - \{-1\}$ d) $R - \{0\}$

135. If $f(x)$ is continuous function and $g(x)$ be discontinuous, then
a) $f(x) + g(x)$ must be continuous
b) $f(x) + g(x)$ must be discontinuous
c) $f(x) + g(x)$ for all x
d) None of these

136. The value of k for which the function
$f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, x \neq 0 \\ k \quad x = 0 \end{cases}$ is continuous at $a = 0$, is
a) $k = 0$ b) $k = 1$ c) $k = -1$ d) None of these

137. The following functions are differentiable on $(-1, 2)$
a) $\int_x^{2x} (\log t)^2 dt$ b) $\int_x^{2x} \frac{\sin t}{t} dt$ c) $\int_x^{2x} \frac{1 - t + t^2}{1 + t + t^2} dt$ d) None of these

138. Let $f(x)$ be defined for all $x > 0$ and be continuous. Let $f(x)$ satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and $f(e) = 1$. Then,
a) $f(x)$ is bounded b) $f\left(\frac{1}{x}\right) \to 0$ as $x \to 0$ c) $xf(x) \to 1$ as $x \to 0$ d) $f(x) = \ln x$

139. Let $f(x) = \begin{cases} \int_0^x \{5 + |1 - t|\} dt, \text{if } x > 2 \\ 5x + 1, \text{if } x \leq 2 \end{cases}$
a) $f(x)$ is continuous at $x = 2$
b) $f(x)$ is continuous but not differentiable at $x = 2$
c) $f(x)$ is everywhere differentiable
d) The right derivative of $f(x)$ at $x = 2$ does not exist

140. Define f on R into itself by
$f(x) = \begin{cases} x \sin \frac{1}{x}, \text{ when } x \neq 0 \\ 0, \text{ when } x = 0 \end{cases}$, then
a) f is continuous at 0 but not differentiable at 0
b) f is both continuous and differentiable at 0
c) f is differentiable but not continuous at 0
d) None of the above

141. If $f(x) = x \sin\left(\frac{1}{x}\right), x \neq 0$, then the value of function at $x = 0$, so that the function is continuous at $x = 0$ is
a) 1 b) -1 c) 0 d) Indeterminate

142. The number of points at which the function $f(x) = (|x - 1| + |x - 2| + \cos x)$ where $x \in [0, 4]$ is not continuous, is
a) 1 b) 2 c) 3 d) 0

143. The function $f(x) = \frac{2x^2+7}{x^3+3x^2-x-3}$ is discontinuous for
a) $x = 1$ only
b) $x = 1$ and $x = -1$ only
c) $x = 1, x = -1, x = -3$ only
d) $x = 1, x = -1, x = -3$ and other values of x

144. The value of the derivative of $|x - 1| + |x - 3|$ at $x = 2$ is
a) 2 b) 1 c) 0 d) -2

145. Let $[x]$ denotes the greatest integer less than or equal to x and $f(x) = [\tan^2 x]$. Then,
a) $\lim_{x \to 0} f(x)$ does not exist
b) $f(x)$ is continuous at $x = 0$

c) $f(x)$ is not differentiable at $x = 0$

d) $f'(0) = 1$

146. Let $f(x)$ be an even function. Then $f'(x)$

a) Is an even function b) Is an odd function c) May be even or odd d) None of these

147. If $f(x) = \{|x| - |x - 1\}^2$, then $f'(x)$ equals

a) 0 for all x

b) $2\{|x| - |x - 1|\}$

c) $\begin{cases} 0 \text{ for } x < 0 \text{ and for } x > 1 \\ 4(2x - 1) \text{for } 0 < x < 1 \end{cases}$

d) $\begin{cases} 0 \text{ for } x < 0 \\ 4(2x - 1) \text{for } x > 0 \end{cases}$

148. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$. Then, which one of the following is true?

a) f is differentiable at $x = 1$ but not at $x = 0$

b) f is neither differentiable at $x = 0$ nor at $x = 1$

c) f is differentiable at $x = 0$ and at $x = 1$

d) f is differentiable at $x = 0$ but not at $x = 1$

149. If $f(x) = \begin{cases} \frac{\log(1+ax)-\log(1-bx)}{x}, x \neq 0 \\ k, \ x = 0 \end{cases}$ and $f(x)$ is continuous at $x = 0$, then the value of k is

a) $a - b$ b) $a + b$ c) $\log a + \log b$ d) None of these

150. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0, n > 0$, and let p be the left hand derivative of $|x - 1|$ at $x = 1$. If $\lim_{x\to1^+} g(x) = p$, then

a) $n = 1, m = 1$ b) $n = 1, m = -1$ c) $n = 2, m = 2$ d) $n > 2, m = n$

151. The value of f at $x = 0$ so that function $f(x) = \frac{2^x - 2^{-x}}{x}, x \neq 0$ is continuous at $x = 0$, is

a) 0 b) $\log 2$ c) 4 d) $\log 4$

152. If $f(x) = \begin{cases} \frac{1-\cos x}{x}, x \neq 0 \\ k, \ x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 0 b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $-\frac{1}{2}$

153. If $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}(\log_e a)^n, a > 0, a \neq 0$, then at $x = 0, f(x)$ is

a) Everywhere continuous but not differentiable

b) Everywhere differentiable

c) Nowhere continuous

d) None of these

154. For the function $f(x) = \frac{e^{1/x}-1}{e^{1/x}+1}, x = 0$, which of the following is correct?

a) $\lim_{x\to0} f(x)$ does not exist

b) $\lim_{x\to0} f(x) = 1$

c) $\lim_{x\to0} f(x)$ exists but $f(x)$ is not continuous at $x = 0$

d) $f(x)$ is continuous at $x = 0$

155. The function $f(x) = |\cos x|$ is

a) Everywhere continuous and differentiable

b) Everywhere continuous and but not differentiable at $(2n + 1)\,\pi/2, n \in Z$

c) Neither continuous nor differentiable at $(2n + 1)\,\pi/2, n \in Z$

d) None of these

156. If $f(x)=\begin{cases}\frac{(4^x-1)^3}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x^2}{3}\right)}, x\neq 0\\ 9(\log 4)^3, x=0\end{cases}$ is continuous function at $x=0$, then the value of a is equal to

a) 3 b) 1 c) 2 d) 0

157. Let $f(x)=\frac{(e^x-1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{4}\right)}$ for $x\neq 0$ and $f(0)=12$. If f is continuous at $x=0$, then the value of a is equal to

a) 1 b) -1 c) 2 d) 3

158. If $f(x)=\begin{cases}1 & \text{for } x<0\\ 1+\sin x & \text{for } 0\leq x\leq \pi/2\end{cases}$, then at $x=0$, the derivative $f'(x)$ is

a) 1 b) 0 c) Infinite d) Does not exist

159. If $f(x)=\begin{cases}x^2\sin\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$, then

a) f and f' are continuous at $x=0$

b) f is derivable at $x=0$ and f' is continuous at $x=0$

c) f is derivable at $x=0$ and f' is not continuous at $x=0$

d) f' is derivable at $x=0$

160. $f(x)=|[x]+x|$ in $-1<x\leq 2$ is

a) Continuous at $x=0$

b) Discontinuous at $x=1$

c) Not differentiable at $x=2,0$

d) All the above

161. Let $f(x)=\frac{\sin 4\pi[x]}{1+[x]^2}$, where $[x]$ is the greatest integer less than or equal to x, then

a) $f(x)$ is not differentiable at some points

b) $f'(x)$ exists but is different from zero

c) $f'(x)=0$ for all x

d) $f'(x)=0$ but f is not a constant function

162. If $f(x)=\begin{cases}mx+1, & x\leq\frac{\pi}{2}\\ \sin x+n, & x>\frac{\pi}{2}\end{cases}$ is continuous t $x=\frac{\pi}{2}$, then

a) $m=1, n=0$ b) $m=\frac{n\pi}{2}+1$ c) $n=m\frac{\pi}{2}$ d) $m=n=\frac{\pi}{2}$

163. If $f(x)=\begin{cases}\frac{\sin 5x}{x^2+2x}, & x\neq 0\\ k+\frac{1}{2}, & x=0\end{cases}$ is continuous at $x=0$, then the value of k is

a) 1 b) -2 c) 2 d) $\frac{1}{2}$

164. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x=c$, then $g'(f(c))$ equals

a) $f'(c)$ b) $\frac{1}{f'(c)}$ c) $f(c)$ d) None of these

165. Let $f(x)=\begin{cases}\frac{\sin\pi x}{5x}, & x\neq 0\\ k, & x=0\end{cases}$, if $f(x)$ is continuous at $x=0$, then k is equal to

a) $\frac{\pi}{5}$ b) $\frac{5}{\pi}$ c) 1 d) 0

166. The value of k which makes $f(x)=\begin{cases}\sin(1/k), & x\neq 0\\ k, & x=0\end{cases}$ continuous at $x=0$ is

a) 8 b) 1 c) -1 d) None of these

167. If $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ exists finitely, then

a) $\lim_{x\to c} f(x)=f(c)$

b) $\lim_{x\to c} f'(x)=f'(c)$

c) $\lim_{x\to c} f(x)$ does not exist

d) $\lim_{x\to c} f(x)$ may or may not exist

168. Let $f(x)$ be a function satisfying $f(x+y)=f(x)+f(y)$ and $f(x)=x\,g\,(x)$ for all $x,y\in R$, where $g(x)$ is continuous. Then,

a) $f'(x)=g'(x)$ b) $f'(x)=g(x)$ c) $f'(x)=g(0)$ d) None of these

169. Let f and g be differentiable functions satisfying $g'(a)=2, g(a)=b$ and $fog=I$ (identify function). Then, $f'(b)$ is equal to

a) 1/2 b) 2 c) 2/3 d) None of these

170. The function $f: R/\{0\}\to R$ given by

$f(x)=\frac{1}{x}-\frac{2}{e^{2x}-1}$

Can be made continuous at $x=0$ by defining $f(0)$ as function

a) 2 b) -1 c) 0 d) 1

171. If $f(x)=\begin{cases}3, & x<0\\ 2x+1, & x\geq 0\end{cases}$, then

a) Both $f(x)$ and $f(|x|)$ are differentiable at $x=0$

b) $f(x)$ is differentiable but $f(|x|)$ is not differentiable at $x=0$

c) $f(|x|)$ is differentiable but $f(x)$ is not differentiable at $x=0$

d) Both $f(x)$ and $f(|x|)$ are not differentiable at $x=0$

172. Let $f(x)=[2x^3-5]$, [] denotes the greatest integer function. Then number of points (1, 2) where the function is discontinuous, is

a) 0 b) 13 c) 10 d) 3

173. A function $f: R\to R$ satisfies the equation $f(x+y)=f(x)f(y)$ for all $x,y\in R$ and $f(x)\neq 0$ for all $x\in R$. If $f(x)$ is differentiable at x $=0$ and $f'(0)=2$, then $f'(x)$ equals

a) $f(x)$ b) $-f(x)$ c) $2f(x)$ d) None of these

174. The set of points, where $f(x)=\frac{x}{1+|x|}$ is differentiable, is

a) $(-\infty,-1)\cup(-1,\infty)$ b) $(-\infty,\infty)$ c) $(0,\infty)$ d) $(-\infty,0)\cup(0,\infty)$

175. Consider $f(x)=\begin{cases}\frac{x^2}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}$

a) $f(x)$ is discontinuous everywhere

b) $f(x)$ is continuous everywhere

c) $f'(x)$ exists in $(-1,1)$

d) $f'(x)$ exists in $(-2,2)$

176. A function f on R into itself is continuous at a point a in R, iff for each $\in>0$, there exists, $\delta>0$ such that

a) $|f(x)-f(a)|<\in\Rightarrow|x-a|<\delta$ b) $|f(x)-f(a)|>\in\Rightarrow|x-a|>\delta$

c) $|x-a|>\delta|f(x)-f(a)|>\in$ d) $|x-a|<\delta|f(x)-f(a)|<\in$

177. If a function $f(x)$ is given by $f(x)=\frac{x}{1+x}+\frac{x}{(x+1)(2x+1)}+\frac{x}{(2x+1)(3x+1)}+\cdots\infty$ then at $x=0, f(x)$

a) Has no limit

b) Is not continuous

c) Is continuous but not differentiable

d) Is differentiable

178. A function $f(x)$ is defined as fallows for real x,

$f(x)=\begin{cases}1-x^2, \text{for } x<1\\ 0, \quad \text{for } x=1\\ 1+x^2, \text{for } x>1\end{cases}$ Then,

a) $f(x)$, is not continuous at $x=1$

b) $f(x)$ is continuous but not differentiable at $x=1$

c) $f(x)$ is both continuous and differentiable at $x=1$

d) None of the above

179. $f(x)=\begin{cases}\frac{\sin 3x}{\sin x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous, if k is

a) 3 b) 0 c) -3 d) -1

180. If a function $f(x)$ is defined as $f(x)=\begin{cases}\frac{x}{\sqrt{x^2}}, x\neq 0\\ 0, \ x=0\end{cases}$ then

a) $f(x)$ is continuous at $x=0$ but not differentiable at $x=0$
b) $f(x)$ is continuous as well as differentiable at $x=0$
c) $f(x)$ is discontinuous at $x=0$
d) None of these

181. $f(x)=\begin{cases}2x, & x<0\\ 2x+1, & x\geq 0\end{cases}$. Then

a) $f(x)$ is continuous at $x=0$
b) $f(|x|)$ is continuous at $x=0$
c) $f(x)$ is discontinuous at $x=0$
d) None of the above

182. The function $f(x)=\max[(1-x),(1+x),2], x\in(-\infty,\infty)$ is

a) Continuous at all points
b) Differentiable at all points
c) Differentiable at all points except at $x=1$ and $x=-1$
d) None of the above

183. If $f(x)=\begin{cases}\frac{x-1}{2x^2-7x+5}, & \text{for } x\neq 1\\ -\frac{1}{3}, & \text{for } x=1\end{cases}$, then $f'(1)$ is equal to

a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) $-\frac{1}{3}$ d) $\frac{1}{3}$

184. The function $f(x)=e^{-|x|}$ is

a) Continuous everywhere but not differentiable at $x=0$
b) Continuous and differentiable everywhere
c) Not continuous at $x=0$
d) None of the above

185. Let [ ] denotes the greatest integer function and $f(x)=[\tan^2 x]$. Then,

a) $\lim_{x\to 0} f(x)$ does not exist
b) $f(x)$ is continuous at $x=0$
c) $f(x)$ is not differentiable at $x=0$
d) $f(x)=1$

186. The function $f(x)=|x|+|x-1|$, is

a) Continuous at $x=1$, but not differentiable
b) Both continuous and differentiable at $x=1$
c) Not continuous at $x=1$
d) None of these

187. If $f(x)=\frac{\log_e(1+x^2\tan x)}{\sin x^3}, x\neq 0$, is to be continuous at $x=0$, then $f(0)$ must be defined as

a) 1 b) 0 c) $\frac{1}{2}$ d) -1

188. Let $f(x+y)=f(x)f(y)$ and $f(x)=1+(\sin 2x)g(x)$ where $g(x)$ is continuous. Then, $f'(x)$ equals

a) $f(x)g(0)$ b) $2f(x)g(0)$ c) $2g(0)$ d) None of these

189. Let $f(x+y)=f(x)f(y)$ and $f(x)=1+xg(x)G(x)$, where $\lim_{x\to 0} g(x)=a$ and $\lim_{x\to 0} G(x)=b$. Then $f'(x)$ is equal to

a) $1+ab$ b) ab c) a/b d) None of these

190. Let $f(x)=\begin{cases}\sin x, & \text{for } x\geq 0\\ 1-\cos x, & \text{for } x\leq 0\end{cases}$ and $g(x)=e^x$. Then, $(gof)'(0)$ is

a) 1 b) -1 c) 0 d) None of these

191. Let $f(x)=[x]$ and $g(x)=\begin{cases}0, & x\in Z\\ x^2, & x\in R-Z\end{cases}$ Then, which one of the following is incorrect?

a) $\lim_{x\to 1} g(x)$ exists, but $g(x)$ is not continuous at $x = 1$
b) $\lim_{x\to 1} f(x)$ does not exist and $f(x)$ is not continuous at $x = 1$
c) gof is continuous for all x
d) fog is continuous for all x

192. If $f(x) = \begin{cases} \frac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, x \neq 0 \\ k, \ x = 0 \end{cases}$ is continuous at $x = 0$, then k equals

a) $16\sqrt{2}\log 2 \log 3$ b) $16\sqrt{2}\ln 6$ c) $16\sqrt{2}\ln 2 \ln 3$ d) None of these

193. Let $f(x)$ be a function such that $f(x+y) = f(x) + f(y)$ and $f(x) = \sin x\, g(x)$ for all $x, y \in R$. If $g(x)$ is a continuous function such that $g(0) = k$, then $f'(x)$ is equal to

a) k b) kx c) $kg(x)$ d) None of these

194. If $f'(a) = 2$ and $f(a) = 4$, then $\lim_{x\to a} \frac{xf(a) - af(x)}{x-a}$ equals

a) $2a - 4$ b) $4 - 2a$ c) $2a + 4$ d) None of these

195. If $f(x) = \begin{cases} ax^2 - b, |x| < 1 \\ \frac{1}{|x|}, \ |x| \geq 1 \end{cases}$ is differentiable at $x = 1$, then

a) $a = \frac{1}{2}, b = -\frac{1}{2}$ b) $a = -\frac{1}{2}, b = -\frac{3}{2}$ c) $a = b = \frac{1}{2}$ d) $a = b = -\frac{1}{2}$

196. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, \ x < 4 \\ a + b, \ x = 4 \\ \frac{x-4}{|x-4|} + b, \ x > 4 \end{cases}$

Then, $f(x)$ is continuous at $x = 4$, when

a) $a = 0, b = 0$ b) $a = 1, b = 1$ c) $a = -1, b = 1$ d) $a = 1, b = -1$

197. If $f(x) = \begin{cases} x - 5, \ \text{for } x \leq 1 \\ 4x^2 - 9, \ \text{for } 1 < x < 2 \\ 3x + 4, \ \text{for } x \geq 2 \end{cases}$, then $f'(2^+)$ is equal to

a) 0 b) 2 c) 3 d) 4

198. Let $f(x) = \begin{cases} \frac{\tan x - \cot x}{x - \frac{\pi}{4}}, \ x \neq \frac{\pi}{4} \\ a, \ x = \frac{\pi}{4} \end{cases}$ the value of a so that $f(x)$ is continuous at $x = \frac{\pi}{4}$ is

a) 2 b) 4 c) 3 d) 1

199. Let$f(x) = [x^3 - x]$, where $[x]$the greatest integer function is. Then the number of points in the interval (1, 2), where function is discontinuous is

a) 4 b) 5 c) 6 d) 7

200. The value of $f(0)$, so that the function

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}}$$

Becomes continuous for all x, is given by

a) $a^{3/2}$ b) $a^{1/2}$ c) $-a^{1/2}$ d) $-a^{3/2}$

201. Let $f(x) = \begin{cases} \frac{1}{|x|} \text{ for } |x| \geq 1 \\ ax^2 + b \text{ for } |x| < 1 \end{cases}$

If $f(x)$ is continuous and differentiable at any point, then

a) $a = \frac{1}{2}, b = -\frac{3}{2}$ b) $a = -\frac{1}{2}, b = \frac{3}{2}$ c) $a = 1, b = -1$ d) None of these

202. If $f(x) = \begin{cases} x^p \cos\left(\frac{1}{x}\right), x \neq 0 \\ 0, \ x = 0 \end{cases}$ is differentiable at $x = 0$, then

a) $p < 0$ b) $0 < p < 1$ c) $p = 1$ d) $p > 1$

203. The function $f(x) = [x] \cos\left[\frac{2x-1}{2}\right]\pi$ where [.] denotes the greatest integer function, is discontinuous at

a) All x
b) No x
c) All integer points
d) x which is not an integer

204. Let $f(x)$ be a function satisfying $f(x+y)=f(x)f(y)$ for all $x,y\in R$ and $f(x)=1+x\,g\,(x)$ where $\lim_{x\to 0} g(x)=1$. Then, $f'(x)$ is equal to
a) $g'(x)$
b) $g(x)$
c) $f(x)$
d) None of these

205. If $f(x)=|\log|x||$, then
a) $f(x)$ is continuous and differentiable for all x in its domain
b) $f(x)$ is continuous for all x in its domain but not differentiable at $x=\pm 1$
c) $f(x)$ is neither continuous nor differentiable at $x=\pm 1$
d) None of the above

206. If $f(x)=\begin{cases}1+x, & 0\le x\le 2\\ 3-x, & 2<x\le 3\end{cases}$ then the set of points of discontinuity of $g(x)=fof(x)$, is
a) {1, 2}
b) {0, 1, 2}
c) {0, 1}
d) None of these